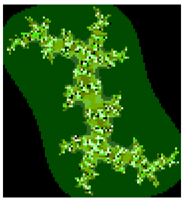

FRACTALS AND SCALE

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Introduction

How long is the coast-line of Great Britain? At first sight this question may seem trivial. Given a map one can sit down with a ruler and soon come up with a value for the length. The problem is that repeating the operation with a larger scale map yields a greater estimate of the length (Fig. 1). If we actually went to the coast and measured them directly, then still greater estimates would result. It turns out that as the scale of measurement decreases the estimated length increases without limit. Thus, if the scale of the (hypothetical) measurements were to be infinitely small, then the estimated length would become infinitely large! Lewis Fry Richardson (quoted in Mandelbrot, 1983) noted this dependence of measured length to the measuring scale used.

In discussing measurement, scale can be characterized in terms of a measuring stick of a particular length: the finer the scale, the shorter the stick. Thus at any particular scale, we can think of a curve as being represented by a sequence of sticks (Fig. 1), all of the appropriate length, joined end-to-end. Clearly, any feature shorter than the stick will vanish from a map constructed in this way. Of course, no one actually makes maps by laying sticks on the ground, but the stick analogy reflects the sorts of distortions that are inevitably produced by the limited resolution of aerial photographs, by the length of survey transects, or by the thickness of the pens used in drafting. There is an analogy here, too, with the accuracy or frequency with which any sort of biological measurements are made.

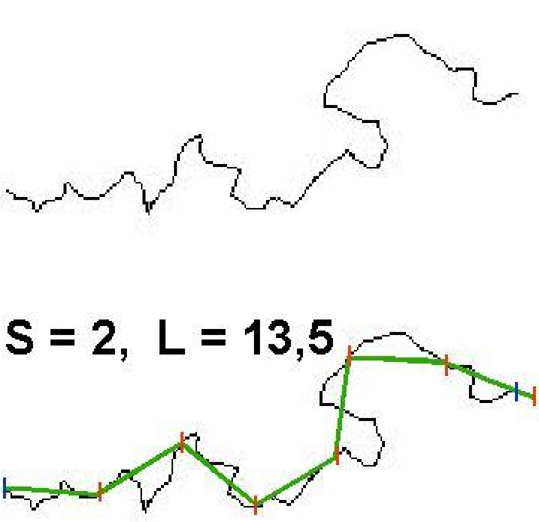

Fig. 1 Using sticks of different size *S* to estimate the length *L* of a coastline.

The dependence of length (or area) measurements on scale poses great problems for biologists who need to use the results. For example, lakes that have a very convoluted shoreline are known to offer more area of shallows in relation to their total surface area, and thus support richer communities of plant and animal life. Attempts to characterize shore-line communities in terms of indexes that relate water surface to shoreline length have been frustrated by problems of scale.

Mandelbrot proposed the idea of a **fractal** (short for "fractional dimension") as a way to cope with problems of scale in the real world. He defined a *fractal* to be any curve or surface that is independent of scale. This property, referred to as *self-similarity*, means that any portion of the curve, if blown up in scale, would appear identical to the whole curve. Thus the transition from one scale to another can be represented as iterations of a scaling process (e.g. Fig. 2).

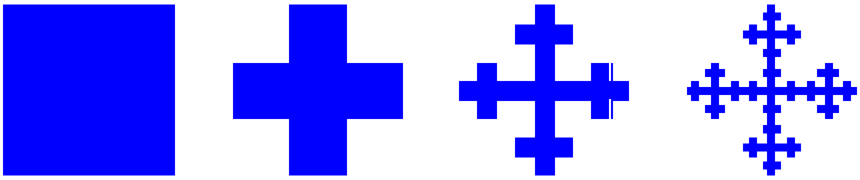

Fig. 2. Forming a cross by iteration of a simple procedure.

An important difference between fractal curves and the idealized curves that are normally applied to natural processes is that fractals are nowhere differentiable. That is, although they are continuous (smooth), they are "kinked" everywhere. Fractals can be characterized by the way in which representation of their structure changes with changing scale.

Fig. 3. A fractal fern.

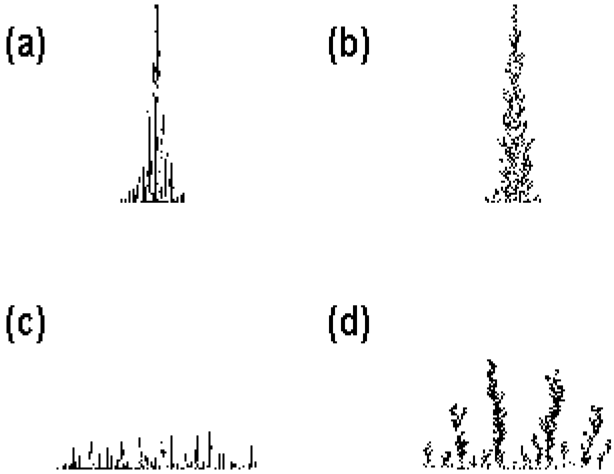

Fig. 4. Structures arising from brownian motion of falling particles.

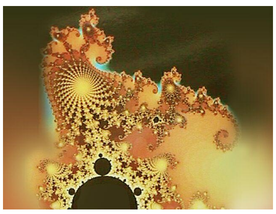

Fig. 5. Mandelbrot´s Fractal

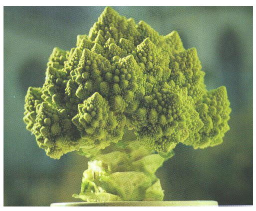

Fig. 6. Brocolis Romanesco

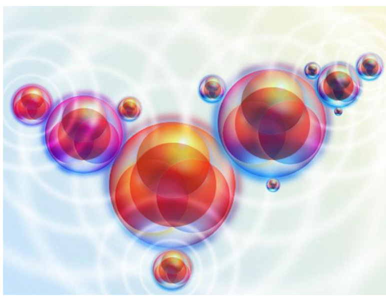

Fig. 7 Enterprise´s Fractal Structure (created by Júlio Tôrres)